Controlling the Evolution of a Quantum System with Dynamical Decoupling Methods

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Abstract

This thesis discusses ways to manipulate the dynamics of chains of spins-½. The evolution of these systems can be altered by applying sequences of magnetic pulses to specific sites within the chain. The control of the system evolution is interesting, for example, in relation to the development of quantum computers. Classical computers represent information by strings of binary digits, or bits. Quantum computing would be based on quantum bits, or qubits, which can be in any superposition of the classical bit's two possible values. As a result of this superposition, a string of qubits can hold more information than a string of classical bits. Quantum computers would be at least as fast as classical computers on some classes of problems and would speed up finding solutions for many other problems. However, challenges lie ahead for the pioneers of quantum information before we can usefully take advantage of the power of quantum systems. One difficulty is to create a physical quantum system with which scientists can implement the theories that have developed around quantum computing. Another main challenge is the presence of decoherence, where quantum systems are easily disturbed by their environment and the information stored in the quantum states can leak out. This thesis will examine a promising method to prevent decoherence by decoupling the system from its environment with a sequence of magnetic pulses. These so-called quantum control methods have also been used to freeze the system evolution by cancelling out its internal interactions. But here, instead of preventing decoherence or freezing the system evolution, the application of these methods is extended to analyze how they can be used to induce a desired dynamics in the quantum systems under investigation.

Introduction

Quantum physics is the branch of physics that deals with several, sometimes counterintuitive phenomena, such as the wave-particle duality, the discretization of energy, the zero-point-energy. In general it is used in the study of the properties of energy and matter on a very small scale, although some properties of mesoscopic and macroscopic systems can only be understood with quantum mechanics. In the classical domain, a system has a definite state at any one time. In contrast, quantum systems acquire a definite state only after a measurement. Before the measurement they may be in a quantum superposition and then they are described as having probabilities of being in certain states. For example, a classical computer stores information in bits. A bit is either 0 or 1, that is, it will always be in one or the other state. In contrast, in a quantum computer, information is stored in quantum bits, or qubits. Each qubit can have a value of 0, 1, or any of the infinite possible superpositions of 0 and 1, as described by

$$|\psi\rangle = \alpha|1\rangle + \beta|0\rangle, \tag{1}$$

Above $|\psi\rangle$ is called the wave function of the system, it is a mathematical tool used in quantum mechanics to describe the state of the system. α and β are complex constants whose absolute squares sum to 1; $|\alpha|^2$ gives the probability of finding the qubit in state $|1\rangle$ after a measurement and $|\beta|^2$ gives the probability of finding it in state $|0\rangle$.

There are different proposals for the physical realization of a qubit. It could correspond, for example, to the ground and excited state of an electron, the presence and absence of a particle, the vertical and horizontal polarization of a photon. Another alternative is a spin-½ in a magnetic field, which is also a binary quantum system. Here, we may associate spin up with the state $|1\rangle$ and spin down with the state $|0\rangle$. This thesis focuses on a one-dimensional spin-½ system which consists of a chain of spins-½ in a magnetic field in the *z*-direction. Each spin in the chain can exist as up or down and nearby spins can have an effect on each other.

The state of a quantum system evolves over time according to:

$$\Psi(t) = U(t)\Psi(0) \tag{2}$$

where Ψ(0) is the initial state and U(t) is the propagator. If the system Hamiltonian, H_0, is time independent, the propagator is given by

$$U(t) = e^{-iH_0 t/\hbar} \tag{3}$$

The changes that occur in the system depend on the system's Hamiltonian which describes its total energy. Each system has its own intrinsic Hamiltonian and propagator. The goal in this thesis is to change the propagator and force the system to evolve as desired. To accomplish the goal, sequences of very strong magnetic fields which rotate the spins will be applied to generate a desired effective propagator.

This so-called method of quantum control has long been employed in nuclear magnetic resonance (NMR) spectroscopy [1,2]. There, the Hamiltonian of nuclear spin systems is modified with sequences of radio-frequency pulses, which constantly rotate the spins and remove or rescale selected terms of the original Hamiltonian. More recently, this idea appeared in the context of quantum information, first as an attempt to fight the effects of decoherence [3-5] and then as a method to freeze the evolution of the system [6-9]. Realistic systems are never isolated and always interact with their environments. This interaction causes the disappearance of quantum superpositions, which is the main ingredient for quantum computers. By applying a sequence of pulses to the system, we may, in principle, cancel out the interaction with the environment. This procedure became known as dynamical decoupling method [3-5]. This technique may also be extended to remove unwanted internal interactions between the qubits and prevent a certain initial state to change in time [6-9].

In this thesis, the effects of the environment are neglected and we focus on the dynamics of an isolated spin-½ chain. Different possible scenarios where unwanted terms are present in the Hamiltonian describing the system are considered and we study how to eliminate them by applying sequences of control pulses. In short, the goal is to construct sequences of pulses capable of modifying the dynamics of the system according to our needs [10].

1. System Model

We study a one-dimensional spin-½ system with open boundary conditions described by the Hamiltonian

$$H_0 = H_z + \beta_1 H_{NN} + \beta_2 H_{NNN} \tag{4a}$$

$$H_z = \sum_{n=1}^{L} \varepsilon_n S_n^z \tag{4b}$$

$$H_{NN} = \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+1}^z + J (S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) \right] \tag{4c}$$

$$H_{NNN} = \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+2}^z + J (S_n^x S_{n+2}^x + S_n^y S_{n+2}^y) \right] \tag{4d}$$

Above, we set $\hbar = 1$, L is the total number of spins in the chain and $S_n^{x,y,z} = \sigma_n^{x,y,z} / 2$ are the spin operators at site *n*, $\sigma_n^{x,y,z}$ being the Pauli matrices. In the one-body term, H_z,

ε_n is the Zeeman splitting of the n^{th} spin in the chain, determined by a static magnetic field in the *z*-direction. A spin pointing up is referred to here as an excitation, since according to H_z it has energy higher than a spin pointing down. $S_n^z S_{n+1}^z$ and $S_n^z S_{n+2}^z$ are the nearest-neighbor and next-nearest-neighbor Ising interactions, respectively, J_z being the strength of the Ising interaction. The flip-flop terms $S_n^x S_{n+1}^x + S_n^y S_{n+1}^y$ and $S_n^x S_{n+2}^x + S_n^y S_{n+2}^y$ have strength J; they hop the excitations along the chain. The ratio between J_z and J defines the anisotropy parameter, Δ. β_1 is the strength of the nearest neighbor terms, β_2 is the strength of the next-nearest neighbor terms. The ratio between β_2 and β_1 is α.

2. Dynamical Decoupling Method

The spin- ½ chains experience propagation in time according to the above Eq. (2). Pulsing the system adds a time dependent control Hamiltonian $H_c(t)$ to the original time independent Hamiltonian, H_0, of the system and the propagator becomes

$$U(t) = \mathrm{T}\exp(-i\int_0^t [H_0 + H_c(u)]du) \tag{5}$$

where T stands for time-ordering and the equation is written from right to left in order of the earlier time. Eq. (5) can be rewritten as (see details in Refs.[1, 2])

$$\begin{aligned} U(t) &= U_c(t)\mathrm{T}\exp(-i\int_0^t [U_c^{-1}(u)H_0 U_c(u)]du) \\ U_c(t) &= \mathrm{T}\exp(-i\int_0^t H_c(u)du) \end{aligned} \qquad \text{(6) and (6a)}$$

where $U_c(t)$ is the control propagator.

We use cyclic control sequences with cycle time T_c. These sequences are designed so that the control propagator and control Hamiltonian are periodic:

$$\begin{aligned} U_c(t+nT_c) &= U_c(t) \\ H_c(t+nT_c) &= H_c(t) \end{aligned} \qquad \text{(7) and (7a)}$$

where n is a natural number.

The control propagator starts out as the identity, $U_c(0) = \mathbf{1}$ so it follows that the control propagator becomes the identity again after every cycle. The general propagator at T_c is then

$$U(T_c) = \mathrm{T}e^{-i\int_0^{T_C}[U_c^{-1}(u)H_0U_c(u)]du} \qquad (8)$$

The propagator that is obtained from the next cycles of pulses applied to the system will be

$$U(nT_c) = U(T_c)^n, \qquad \text{(8a)}$$

so we know what the future evolution of the system will be just by looking at the evolution of the first cycle.

In our case, the control operations correspond to strong and instantaneous pulses, P, which is known as bang-bang control. They are applied after time intervals of free evolution, $\tau_{k+1} = t_{k+1} - t_k$, where $k \in \mathbb{N}$ and $t_0 = 0$. For a cycle with m pulses, the evolution operator after one cycle is:

$$U(T_c) = P_m U(t_m, t_{m-1}) P_m U(t_{m-1}, t_{m-2}) \ldots P_2 U(t_2, t_1) P_1 U(t_1, t_0), \qquad \text{(9a)}$$

Since $P^\dagger P = \mathbf{1}$ the equation above can be expanded to

$$U(T_c) = (P_m P_{m-1} ... P_1)(P_{m-1} P_{m-2} ... P_1)^{\dagger} U(t_m, t_{m-1})(P_{m-1} ... P_1)...$$
$$(P_2 P_1)^{\dagger} U(t_3, t_2)(P_2 P_1) P_1^{\dagger} U(t_2, t_1) P_1 U(t_1, 0) \tag{9b}$$

which leads to

$$U(T_c) = \exp[-iH_{m-1}\tau_m]...\exp[-iH_2\tau_3]\exp[-iH_1\tau_2]\exp[-iH_0\tau_1] \tag{9c}$$

where $H_{m-1} = (P_{m-1}...P_1)^{\dagger} H_0 (P_{m-1}...P_1)$.

Using the Baker-Campbell-Hausdorff expansion, we can express the general propagator as a function of the average Hamiltonian, $\bar{H}$, as:

$$U(T_c) = \exp(-i\bar{H}T_c). \tag{10}$$

The lowest order term of the average Hamiltonian is the sum

$$\bar{H}^{(0)} = \sum_{k=0}^{m-1} \frac{\tau_{k+1}}{T_c} H_k, \tag{11}$$

whereas the higher order terms involve commutators of H_k's. We will focus on manipulating the lowest order term of the average Hamiltonian. This is a good initial approximation, since the effects that come from the higher order terms only become significant after a lot of time has passed or when the time between pulses is too large.

In this thesis we consider π- pulses which rotate the spins by 180^{o} around the *x, y* and *z* directions:

$$P_{x,y,z} = \exp[-i\pi \ S_{x,y,z}]. \tag{12}$$

Applying a pulse in a certain direction flips the spins in the directions perpendicular to the applied pulse.

2.1 An example: spin echo

To illustrate the way the pulses affect a system's evolution, let us take a look at the famous case of spin echo. In this case, the original system contains only one spin pointing up in the *z*-direction, so the initial Hamiltonian is $H_0 = \varepsilon S_z$. Our goal here is to freeze the system, so our method will pulse twice in the *x*-direction waiting τ units of time before each pulse. The first pulse flips the spin down, and after the system evolves freely for τ, the second pulse is applied which flips the spin back up. The system comes back to the initial state after this cycle is completed.

The propagator in this case is very simple. After the first cycle of $T_c = 2\tau$,

$$U(T_c = 2\tau) = P_x e^{-i\varepsilon S_z \tau} P_x e^{-i\varepsilon S_z \tau}$$
$$U(2\tau) = \underbrace{e^{-i\pi S_x} e^{-i\pi S_x}}_{-1} e^{+i\pi S_x} e^{-i\varepsilon S_z \tau} e^{-i\pi S_x} e^{-i\varepsilon S_z \tau} \quad \text{(13) and (13a)}$$

In Eq. (13a), the second term of free evolution is sandwiched between P_x and its complex conjugate. This sandwiching of the propagator will change the sign of S_z. The change can be detected by doing a couple of Taylor expansions and manipulations to the sandwich:

$$e^{i\pi S_x} e^{-i\varepsilon S_z \tau} e^{-i\pi S_x} = e^{i\pi S_x}[\mathbf{1} + (-i\varepsilon S_z \tau) + \frac{(-i\varepsilon S_z \tau)^2}{2!} + ...]e^{-i\pi S_x}$$
$$= \mathbf{1}(1 + e^{i\pi S_x}\left(-i\varepsilon S_z \tau\right)e^{-i\pi S_x} - e^{i\pi S_x}\frac{\varepsilon^2\tau^2}{2!}e^{-i\pi S_x}...) \quad (14)$$

The last expression is also the Taylor expansion of $\exp[e^{i\pi S_x}(-i\varepsilon S_z \tau)e^{-i\pi S_x}]$. We can also verify that $e^{-i\pi S_x} = -2iS_x$ by way of a Taylor expansion,

$$\exp(-i\pi S_x) = 1 - \frac{i\pi}{2}(2S_x) + \frac{(-i\pi/2)^2}{2!} + \frac{(-i\pi/2)^3}{3!}(2S_x) + \cdots$$

$$= \cos\frac{\pi}{2} - i\,(2S_x)\sin\frac{\pi}{2}$$

After simplifying, since $4S_xS_zS_x = -S_z$, the sandwich turns into $\exp[+i\varepsilon S_z\tau]$. A sign change is produced in the Hamiltonian for the *z*-term. The propagator becomes $U(2\tau) = (-1)\exp(+i\varepsilon\ S_z\tau)\exp(-i\varepsilon\ S_z\tau)$ which, apart from a sign, is the identity and so the system is frozen.

3. Quantities Measured and Initial State

We can trace the evolution of a system by looking at how its observables change over time. In this work, the observable we will focus on is the local magnetization. Local magnetization measures how many spins are pointing up in the *z* direction in the first half of the chain,

$$< M(t) >=< \Psi(t) | \sum_{n=1}^{L/2} S_n^z |\Psi(t) > \qquad (15)$$

The dynamics depends on the initial state of the system. In the systems we test, we use chains that initially have all the spins in the first half of the chain pointing up and all the others pointing down.

We can also compute a theoretical quantity known as the propagator fidelity. It is a good tool to confirm that our quantum control methods will work to achieve a desired dynamics regardless of the initial state. The propagator fidelity is defined as

$$F_u(t) \equiv \left|\mathrm{Tr}\left[U_W^\dagger(t)U(t)\right]\right| / 2^L \qquad (16)$$

Our control methods are successful when we manage to bring $U(t)$ close to the wanted propagator, $U_w(t)$, and therefore achieve a propagator fidelity close to 1. Results for fidelity are not included in this thesis, but they can be found in reference [10].

4. Numerical Results

We now show how some specific pulse sequences may produce a desired dynamics for the spin-½ chains under investigation.

4.1 A nonintegrable system is forced to behave as an integrable system

Depending on the parameters of the Hamiltonian, the system may be integrable or chaotic. It is known that chaotic systems show diffusive transport behavior, whereas integrable systems show ballistic behavior [11]. Very roughly, it means that the flow of particles, excitations or heat through an integrable chain is much easier and faster than in the chaotic case. In this section we will consider a chaotic system and study how to change its effective propagator so that its evolution becomes close to that of an integrable system.

4.1.1 The disorder case

In the case where $\beta_2 = 0$, chaos may occur in the presence of on-site disorder. When all of the on-site energies in the chain are the same, $\varepsilon_n = \varepsilon$, the system is clean and in this case it is integrable. If there is at least one defect in the system (i.e. one of the on-site energies is different from the rest) the system becomes disordered. Even in the presence of a single defect, the system is chaotic if the defect is away from the borders of the chain and is of the order of J_z [12]. The Hamiltonian of the system we will deal with here contains a single defect in the middle of the chain, on site *L/2+1*, and nearest-neighbor couplings only:

$$H_0 = H_z + H_{NN}$$

$$H_0 = \varepsilon_{L/2+1} S^z_{L/2+1} + \sum_{n \neq L/2+1} \varepsilon S^z_n + \sum_{n=1}^{L-1} \left[J_z S^z_n S^z_{n+1} + J\left(S^x_n S^x_{n+1} + S^y_n S^y_{n+1} \right) \right] \qquad (17)$$

We want to pulse the system with a sequence that will get rid of the effects of the defect and recover the transport behavior of an integrable system. The proposed cycle contains two pulses in the *x*-direction capable of flipping every spin in the chain around the *x*-axis. (Note: each pulse will correspond to two very strong magnetic fields, one in resonance with the spins that have energy splitting ε, which will rotate all of these spins by 180°, and the other to rotate the defect). Each S_y and S_z operator in the Hamiltonian changes sign and therefore the double sign change in the nearest neighbor terms blocks the pulses' effect on the couplings terms. The sign change of H_z remains and after both pulses, this term drops out of the lowest order term in the average Hamiltonian. The scenario is very similar to the spin echo in Sec.2.1, but now for several coupled spins. The propagator after *j* cycles will be

$$\begin{aligned} U(2j\tau) &= P_x e^{-(+H_Z+H_{NN})\tau} P_x e^{-(+H_Z+H_{NN})\tau} \ldots P_x e^{-(+H_Z+H_{NN})\tau} P_x e^{-(+H_Z+H_{NN})\tau} \\ U(2j\tau) &= e^{-i(-H_Z+H_{NN})\tau} e^{-i(+H_Z+H_{NN})\tau} \ldots e^{-i(-H_Z+H_{NN})\tau} e^{-i(+H_Z+H_{NN})\tau}. \end{aligned} \qquad (18)$$

In first order in τ only the coupling terms remain. The first term in the average Hamiltonian of the system becomes $\bar{H}^{(0)} = H_{NN}$. The system will then behave as if its Hamiltonian only contained the nearest neighbor terms. This is illustrated in Fig.1 below. For the pulsed chaotic system, the transport of local magnetization coincides with the one obtained for the integrable chain, where a bouncing behavior is noticed. When small values of τ are used, the agreement may hold for relatively long times, as shown in the figure.

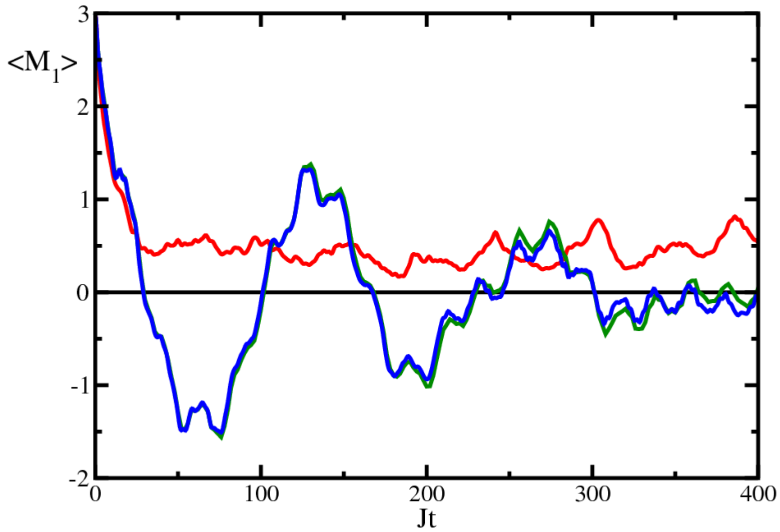

Figure 1. Time evolution of local magnetization for a spin-½ chain with L=12 sites. The initial state is first 6 spins up and last 6 spins down. A defect is present at site 7 and the anisotropy parameter is Δ =1. The cycle time is $T_c = 2\tau$ and the interval between pulses is $\tau = J^{-1}$. The red line is the unpulsed disordered system, blue line is an integrable system with $H_0 = H_{NN}$ and the green line is the pulsed disordered system. The points that make up the curve are drawn after every T_c.

4.1.2 The next nearest neighbor case

Chaos also may originate from the presence of terms that frustrate the system, that is, when $\beta_2 \neq 0$, even when the chain is clean. Since J and J_z are positive, the spins in the chain with nearest-neighbor couplings like to align in an antiferromagnetic configuration. The tendency introduced by the next nearest neighbor term to have also antiferromagnetic alignment between second neighbor brings frustration into the system. When the frustration is strong (i.e. $\alpha = \beta_2 / \beta_1$ is greater than a critical value,) the system becomes chaotic.

We start with a system whose initial Hamiltonian contains both nearest neighbor and next nearest neighbor two-body terms,

$$\begin{aligned} &H_0 = \beta_1 H_{NN} + \beta_2 H_{NNN} \\ &H_0 = \beta_1 \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+1}^z + J\left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y \right) \right] \\ &+\beta_2 \sum_{n=1}^{L-2} \left[J_z S_n^z S_{n+2}^z + J\left(S_n^x S_{n+2}^x + S_n^y S_{n+2}^y \right) \right] \end{aligned} \tag{19}$$

We test to see if we can get rid of the effects of the frustration by applying a specially designed sequence of magnetic pulses to the system, whose cycle contains 8 pulses, as shown in Table 1.

$$P_1 = P_3 = \prod_{k=0}^{\lfloor (L-1)/4 \rfloor} e^{-i\pi S_{1+4k}^x} \prod_{k=0}^{\lfloor (L-2)/4 \rfloor} e^{-i\pi S_{2+4k}^x}$$

$$P_2 = P_4 = \prod_{k=0}^{\lfloor (L-3)/4 \rfloor} e^{-i\pi S_{3+4k}^y} \prod_{k=0}^{\lfloor (L-4)/4 \rfloor} e^{-i\pi S_{4+4k}^y}$$

$$P_5 = P_7 = \prod_{k=0}^{\lfloor (L-2)/4 \rfloor} e^{-i\pi S_{2+4k}^x} \prod_{k=0}^{\lfloor (L-3)/4 \rfloor} e^{-i\pi S_{3+4k}^x}$$

$$P_6 = P_8 = \prod_{k=0}^{\lfloor (L-1)/4 \rfloor} e^{-i\pi S_{1+4k}^y} \prod_{k=0}^{\lfloor (L-4)/4 \rfloor} e^{-i\pi S_{4+4k}^y}$$

Table 1. Pulse sequence applied to get rid of the effects of next-nearest-neighbor couplings.

This sequence is more complex than in the previous case. The pulses are only applied to specific sites in the chain since we want to affect the coupling terms. Table 2 shows how each of the terms in the Hamiltonian are affected as the pulses are applied throughout the cycle.

NN	$0,\tau$	$\tau,2\tau$	$2\tau,3\tau$	$3\tau,4\tau$	$4\tau,5\tau$	$5\tau,6\tau$	$6\tau,7\tau$	$7\tau,8\tau$
$S_{2i+1}^x S_{2i+2}^x$	$+$	$+$	$+$	$+$	$+$	$+$	$-$	$-$
$S_{2i+1}^x S_{2i+2}^x$	$+$	$+$	$-$	$-$	$+$	$+$	$+$	$+$
$S_{2i+1}^x S_{2i+2}^x$	$+$	$+$	$+$	$+$	$+$	$-$	$-$	$+$
$S_{2i+1}^x S_{2i+2}^x$	$+$	$-$	$-$	$+$	$+$	$+$	$+$	$+$
$S_{2i+1}^x S_{2i+2}^x$	$+$	$+$	$+$	$+$	$+$	$-$	$+$	$-$
$S_{2i+1}^x S_{2i+2}^x$	$+$	$-$	$+$	$-$	$+$	$+$	$+$	$+$
NNN	$0,\tau$	$\tau,2\tau$	$2\tau,3\tau$	$3\tau,4\tau$	$4\tau,5\tau$	$5\tau,6\tau$	$6\tau,7\tau$	$7\tau,8\tau$
$S_{2i+1}^x S_{2i+2}^x$	$+$	$+$	$-$	$-$	$+$	$+$	$-$	$-$
$S_{2i+1}^x S_{2i+2}^x$	$+$	$-$	$-$	$+$	$+$	$-$	$-$	$+$
$S_{2i+1}^x S_{2i+2}^x$	$+$	$-$	$+$	$-$	$+$	$-$	$+$	$-$

Table 2. How the pulses change the terms in the frustrated system's Hamiltonian.

At the cycle's end, the contributions from the next nearest neighbor terms are cancelled out from the average Hamiltonian in first order in τ. Additionally, half of the nearest neighbor terms are cancelled out as well. The system evolves more slowly, as if its original Hamiltonian contained the nearest neighbor term divided in half; the dominant term in the average Hamiltonian becomes $\bar{H}^{(0)} = H_{NN}/2$. As illustrated in Figure 2, we see that when we apply this sequence of pulses we obtain the transport behavior of the integrable system, albeit slower. The evolution of the pulsed system matches the desired evolution most closely at the end of each pulsing cycle.

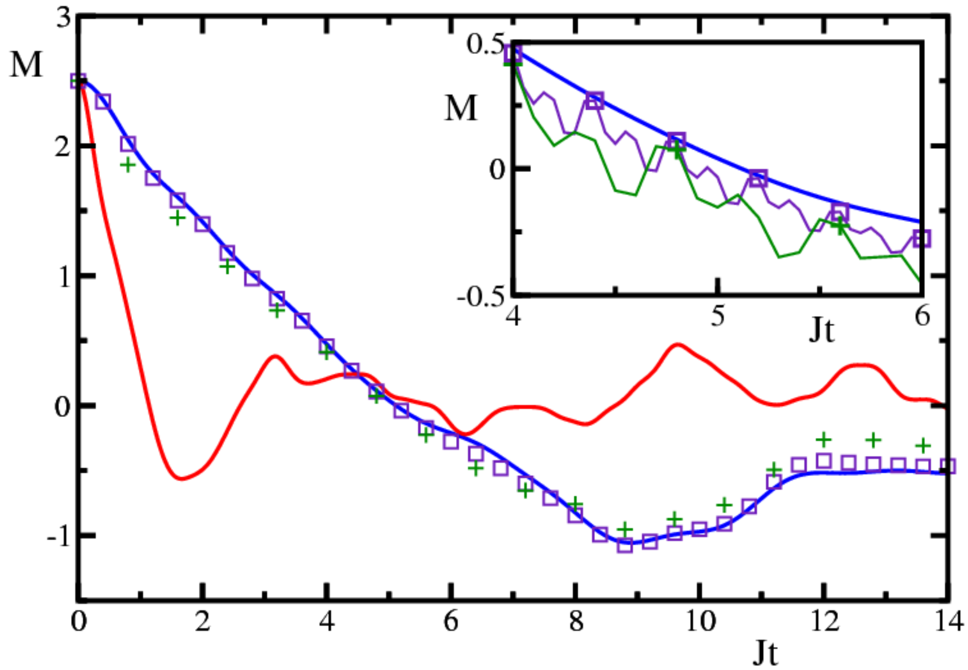

Figure 2. Time evolution of local magnetization for a clean spin-½ chain with Δ=1 and L=10 sites. Main panel: The red line represents the free evolution of a frustrated system with α=1. The blue line is the evolution of a system with α=0 and $H_0 = H_{NN}/2$. The purple markers (square) and the green markers (plus) show the evolution of a frustrated system pulsed at intervals of $\tau = 0.05J^{-1}$ and $\tau = 0.1J^{-1}$ respectively. Only the data at the end of each cycle is shown. Inset: The inset provides the data after each pulse. It shows how the pulsed system's evolution matches the desired evolution the best at the end of the pulsing cycle, which is marked with symbols.

4.2 Gapped vs. gapless systems

In this last case, we consider a clean system with $\beta_2 = 0$ and whose original Hamiltonian contains all of the nearest neighbor coupling terms, both the Ising interaction and the flip-flop term.

$$H_0 = H_{NN}$$
$$H_0 = \sum_{n=1}^{L-1} \left[J_z S_n^z S_{n+1}^z + J\left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y \right) \right] \tag{20}$$

When the anisotropy, $\Delta = J_z / J$, of the system is very strong (i.e. $\Delta > 1$,) the system is in the gapped phase. If the anisotropy is weak (i.e. $\Delta < 1$,) the system is in the gapless phase. (We assume that Δ is always positive). We consider a system initially in the gapless phase. Our goal now is to search for a sequence of bang-bang pulses to control the behavior of the system and make it act as if it was in the gapped phase.

This time, we do not want to completely eliminate the effects of any one term, rather we just want to reduce the weight of the flip flop term with respect to the Ising interaction to achieve gapped behavior. In order to accomplish this, we will vary the time interval between the pulses. The pulses are applied only to the odd spins in the *z*-direction so that the *z*-spin operators are not affected and only the flip flop term changes sign. The system is allowed to evolve freely for τ_1 and is then pulsed. Again the system evolves freely for $\tau_2 < \tau_1$ and then the final pulse of the cycle is applied to the odd spins. The resulting propagator is

$$U(T_c = \tau_1 + \tau_2) = P_z^{odd} e^{-iH_{NN}\tau_2} P_z^{odd} e^{-iH_{NN}\tau_1}$$
$$U(T_c) = e^{-i(\sum_{n=1}^{L-1}\left[J_z S_n^z S_{n+1}^z - J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y)\right])\tau_2} e^{-i(\sum_{n=1}^{L-1}\left[J_z S_n^z S_{n+1}^z + J(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y)\right])\tau_1} \quad (21)$$

and the dominant term of the average Hamiltonian becomes

$$\bar{H}^{(0)} = \sum_{n=1}^{L-1}[\tfrac{J(\tau_1 - \tau_2)}{T_c}(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y) + J_z S_n^z S_{n+1}^z]. \quad (22)$$

When good values are chosen for τ_1 and τ_2, the pulse sequence effectively reduces the contributions of the flip-flop term and boosts the anisotropy parameter,

$$\Delta = \frac{J_z T_c}{J(\tau_1 - \tau_2)} \tag{23}$$

to produce gapped behavior in the system's evolution. This is illustrated in Fig.3. As pictured in the left panel, the pulsed gapless system with anisotropy parameter Δ=½ evolves as if it was a gapped system with Δ=2. Notice in the right panel that using cycles with shorter, rather than longer, intervals between applied pulses brings the system closer to the desired gapped evolution.

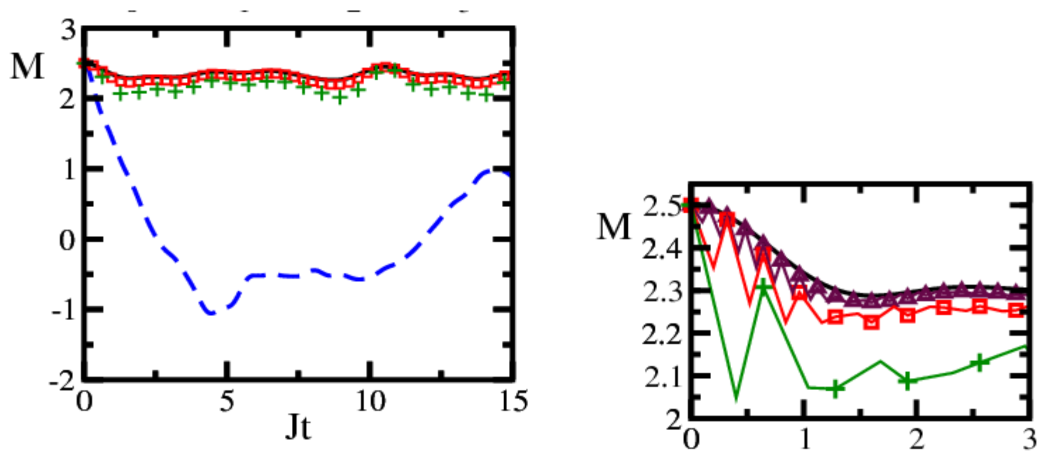

Figure 3. Time evolution of local magnetization for a clean spin-½ chain with L=10 sites. The blue dashed line is a gapless system with anisotropy parameter, Δ=½. The black line is a gapped system with Δ=2. The colored markers are gapless systems that were pulsed at the following intervals; purple: $\tau_1 = 0.01J^{-1}, \tau_2 = 0.006J^{-1}$, red: $\tau_1 = 0.02J^{-1}, \tau_2 = 0.012J^{-1}$ and green $\tau_1 = 0.04J^{-1}, \tau_2 = 0.024J^{-1}$. These intervals satisfy Eq. (20) to bring the anisotropy parameter to 2.

Notice that in the gapped phase and for our particular initial state, the spins in the system change their orientation very slowly; there is little movement as seen in Fig.3. This slow behavior is caused by a strong presence of the Ising term. The Ising interaction adds +*J*Δ/4 units of energy to the system when adjacent spins are aligned in same directions and -*J*Δ/4 units of energy when adjacent spins are aligned antiferromagnetically. As a result, our initial state has a very high energy of *J*

$(L-3)\Delta/4$. In the isolated systems we study, the energy of the system must be maintained overall, so the excitations can only move if spin configurations with energy close to our initial state may be found. In the gapped phase there are very few states that satisfy this requirement, which then limits the movement in the chain caused by the hopping term.

5. Conclusions

In this work, we used quantum control methods to induce specific transport behavior in isolated spin-½ chains. We see that a system that is chaotic due to a defect can be pulsed to behave like an integrable chain. In addition, a chaotic frustrated system can be manipulated to behave like an integrable chain with a slightly more complicated sequence of pulses. We can also use the dynamical decoupling methods to induce a change in behavior between systems in the gapped and gapless phases. Specific strategies that were used to obtain a desired dynamics from a system include pulsing specific sites in the chain and varying the time intervals between pulses. It now remains to study how to realize these sequences experimentally.

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