Lesson 02

Defining functions

Sometimes we may need to use the same function over and over again and it may be handy to give the function a name.

-) Example: we need to compute the position y of an object in free fall at a certain instant of time t and for various initial positions yo and velocities vo.
Choosing a framework where up is positive, g (pointing down) will be negative. We can then define the function, as

```
Clear[y,g];
g = - 9.8;
y[t_, yo_, vo_] := yo + vo*t + (g/2.)*t^2;
```

And compute the position:

```
y[ 1, 50, 10 ]
y[ 2, 50, 10 ]
y[ 2 ,50, - 10]
```

Lists, Table, Manipulating lists

- **List**

```
list = {4,6,2,3,9,7,1};
1/list
list!
3.*list

Total[ list ]

Max[ list ]
Min[ list ]

Sort[list]
Reverse[ Sort[ list ] ]

Length[ list ]
Position[ list, Max[ list ] ]
Position[ list, Last[ list ] ]
```

- **Table (vector)**

The Table function is used to generate lists

-) List of numbers from 5 to 10

```
t1=Table[k, {k, 5, 10}]
MatrixForm[t1]
```

-) List of the five first perfect squares t2=Table[k^2, {k, 1, 5}]

-) List of odd numbers in decreasing order starting from 13
t3=Table[k, {k, 13, 1, -1}]

-) Construct a list of values of y[t_, yo_, vo_] for yo=50, vo=5, and t = 0, 0.5, 1.0, 1.5., 2.0, 2.5, 3.0, 3.5, 4.0
Table[y[0.5 k, 50, 5], {k,0,16}]

■ Working with the list

In a table called "tab", tab[[n]] gives the nth element of the list, and tab[[- n]] gives the nth element from the end of the list. The command Part[tab, n] also has the same role.

evens = Table[k, {k,0,16,2}]
evens[[3]]
Part[evens, 3]
evens[[- 2]]
Part[evens, -2]

Other ways to manipulate a list:

Take[evens, 2]
Take[evens, -3]
Take[evens, {2, 4}]

Drop[evens, 3]
Drop[evens, - 2]

Union[t1, t2]
Intersection[t1, t2]
Join[t1, t2]

Matrices

■ Table (matrix)

A matrix is an array of numbers arranged in rows and columns

m1 = { {1, 2, 3, 4}, {5, 6, 7, 8}, {9, 10, 11, 12} };
MatrixForm[m1]

m2=Flatten[m1]
MatrixForm[m2]

m = Table[Table[a[row, column], {column, 1, 3 }], {row, 1, 4}]
MatrixForm[m]

Matrices can be combined using the operations of addition, subtraction, scalar, and matrix multiplication.
The operation of matrix multiplication is represented by a period (.).

REVIEW (do it in a piece of paper and then compare with *Mathematica*):

-) If you do not remember how to deal with matrices, *Mathematica* can help.
MM = { { m11, m12, m13}, {m21, m22, m23}, {m31, m32, m33} };
PP= { { p11, p12, p13 }, {p21, p22, p23}, {p31, p32, p33} };
u = { u1, u2 , u3 };
MatrixForm[MM + PP]
MatrixForm[MM - PP]
MatrixForm[MM. PP]
MatrixForm[MM.u]

A = { { 1, 2, 3}, {4, 5, 6}, {7, 8, 9} };
B= { { 2, 1, 5 }, {4, 7, 2}, {1, 3, 2} };
MatrixForm[A + B]
MatrixForm[A - B]
MatrixForm[3 A + B]
MatrixForm[A . B]

-) In matrix B, what is the element in the 3rd row and in the 2nd column?
B[[3, 2]]
-) Add the element B in 3rd row and in the 2nd column with the element of A in the second row and in the first column
B[[3, 2]] + A[[2, 1]]
-) Multipply matrix A by the vector vec = { 5, 5, 5 }
B . vec

-) diag=DiagonalMatrix[{ 1,4, 7 }]
MatrixForm[diag]
diag // MatrixForm

-) id=IdentityMatrix[4];
MatrixForm[id]

REVIEW:
VECTORS
u = { u1, u2 , u3 };
v = { v1, v2, v3 };
-) Dot product (example: W=**F.d**)
u.v
-) Cross product (example: **F**=q **vxB**)
Cross[u,v]
-) Outer (tensor product)
ou=Outer[Times, u, v] ;
MatrixForm[ou]

REVIEW:
mat = { {1,2,3}, {4,2,2}, {5,1,7} };
MatrixForm[mat]

-) Transpose[mat]
MatrixForm[Transpose[mat]]

-) Determinant (system of linear equations has solution if det is nonzero, Jacobian in integrals, eigenvalues of a matrix)
Det[mat]

-) mat.mit=**1** implies that *mit* is the inverse matrix of *mat*
A matrix is invertible only if its det is nonzero
Inverse[mat]

-) Tr[mat]

■ Table Form

-) Make a matrix with the values of y[t,50,5] for t = 0, 0.5, 1.0, 1.5., 2.0, ... 4.0, where the first column gives the time and the second the value y of the position.

ty=Table[{0.5 k , y[0.5 k, 50, 5] }, { k, 0, 8}]
TableForm[ty]

form = Table[{ PaddedForm[0.5 k, {3, 1}] , PaddedForm[y[0.5 k, 50, 5], { 7, 3 }] }, { k, 0, 8}]
TableForm[form]
TableForm[form, TableHeadings ->{None, { " time", " position"} }]

ListPlot[ty]
ListPlot[ty, Joined -> True]

Random numbers

Table with 10 random numbers:

-) Integers 0 or 1
Table[RandomInteger[ ] , { k, 1, 10 }]

-) Integers between 2 and 5
Table[RandomInteger[{ 2 , 5 }] , { k, 1, 10 }]

-) Reals between 0 and 1
Table[RandomReal[ ] , { k, 1, 10 }]

-) Reals between 1 and 4
Table[RandomReal[{ 1 , 4 }] , { k, 1, 10 }]

-) Random numbers from a normal (Gaussian, Bell) distribution with mean 0 and standard deviation 1
Table[RandomReal[NormalDistribution[0, 1]], { k, 1, 10 }]